

Distributed Approximate Message Passing for Compressed Sensing

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Abstract—In this paper, an efficient distributed approach for implementing the approximate message passing (AMP) algorithm, named distributed AMP (DAMP), is developed for compressed sensing (CS) recovery in sensor networks with the sparsity K unknown. In the proposed DAMP, distributed sensors do not have to use or know the entire global sensing matrix, and the burden of computation and storage for each sensor is reduced. To reduce communications among the sensors, a new data query algorithm, called global computation for AMP (GCAMP), is proposed. The proposed GCAMP based DAMP approach has exactly the same recovery solution as the centralized AMP algorithm, which is proved theoretically in the paper. The performance of the DAMP approach is evaluated in terms of the communication cost saved by using GCAMP. For comparison purpose, thresholding algorithm (TA), a well known distributed Top-K algorithm, is modified so that it also leads to the same recovery solution as the centralized AMP. Numerical results demonstrate that the GCAMP based DAMP outperforms the Modified TA based DAMP, and reduces the communication cost significantly.

Index Terms—Compressed Sensing, Distributed AMP.

I. INTRODUCTION

Compressed sensing (CS) has wide applications in various areas of signal processing [1]. Due to the curse of dimensionality, it can be highly demanding to perform CS on a single processor. Further, distributed processing has the potential to reduce communications among distributed sensors. Hence, distributed CS (DCS) in sensor networks has become an interesting topic. A general DCS system contains two parts: (1) the local computation performed at each sensor, and (2) the global computation to obtain the estimate of the original sparse signal after sensors exchange the results of local computation. Several distributed approaches based on various CS recovery algorithms were proposed. In [2], a distributed subspace pursuit (DiSP) algorithm was developed to recover joint sparse signals. In DiSP, each sensor needs to store the global sensing matrix, and local computation at each sensor involves optimization and matrix inversion. The computation and memory burden may become very challenging for each sensor in large-scale problems. Further, in DiSP the sparsity K is assumed to be known, which may not be the case in many applications. In [3], an algorithm named D-ADMM based on basis pursuit (BP) was proposed, in which sensors do not have to store the entire global sensing matrix. However, each sensor still needs to solve an optimization problem to get an recovery per iteration, and broadcasts it to its neighbors, which may

induce high communication cost since the recovery in first few iterations is not sparse. Focusing on these problems, a DCS algorithm based on iterative hard thresholding (IHT) named D-IHT was proposed in [4]. In the local computation, each sensor just performs very simple operations such as matrix transpose, addition and multiplication. In the global computation, thresholding algorithm (TA) [5] has been applied, which is a popular method to solve the distributed Top-K problem in the field of database querying, to reduce the amount of messages sent between sensors. Nevertheless, in the D-IHT, the sparsity K was also assumed to be known. Further, the D-IHT requires each local sensor to know certain prior knowledge about the global sensing matrix, such as its L_2 norm. For a certain sensor node (or a fusion center) to know the global sensing matrix to calculate and then broadcast its L_2 norm, each of the rest sensor nodes has to either transmit its local sensing matrix or the seed of its local random number generator used to generate the corresponding local sensing matrix.

In this paper, we do not assume the knowledge of sparsity and hence the IHT cannot be directly applied. Instead, we propose a distributed algorithm based on approximate message passing (AMP) [6], which does not require any prior knowledge of the sparse signal, and has a linear convergence rate [6], [7]. For the proposed distributed AMP (DAMP) approach, we do not assume any prior knowledge of the global sensing matrix. Distributed sensors do not need to store the entire global sensing matrix. In the local computation, each sensor only performs simple matrix operations, and in the global computation per iteration, we propose a new algorithm, Global Computation for AMP (GCAMP), to reduce the amount of data transmitted in the sensor network. To the best of our knowledge, the proposed approach is the first distributed AMP algorithm ever developed.

II. DAMP SYSTEM

A. The Original AMP

A task of CS is to recover a sparse signal $s_0 \in R^N$ from its measurement $y = As_0 + n$, where $A \in R^{M \times N}$ is the sensing matrix and n is an additive noise, by solving the problem:

$$\min_x \frac{1}{2} \|y - Ax\|_2^2 + \lambda \|x\|_1 \quad (1)$$

where $\lambda > 0$ is the regularization parameter. However, λ is not given in most practical cases. AMP is a good solution to the

problem [6] without prior knowledge about K and λ . Starting from $x_0 = 0$ and $z_0 = y$, it recursively gets the new estimate of s_0 as follows:

$$x_{t+1} = \eta_t(x_t + A^T z_t; \tau \sigma_t) \quad (2)$$

$$z_{t+1} = y - A x_{t+1} + \frac{\|x_{t+1}\|_0}{M} z_t \quad (3)$$

where $[\cdot]^T$ denotes the transpose operation, $\|\cdot\|_0$ is the l_0 norm of a vector, $\sigma_t^2 = \frac{\|z_t\|^2}{M}$ [8],

$$\eta_t(x; \beta) = \begin{cases} (|x| - \beta) \text{sgn}(x), & |x| > \beta \\ 0, & |x| \leq \beta \end{cases} \quad (4)$$

and τ is a parameter whose optimal value depends on $\kappa = \frac{M}{N}$ and $\rho = \frac{K}{M}$ [8]. Since K is unknown, a tuning procedure is needed, which will be presented later in this paper, to find a value for τ which is very close to the optimum.

B. The Distributed Framework of AMP

Let us consider a sensor network with P distributed sensors. Each sensor p ($p = 1, \dots, P$) takes a measurement of s_0 as

$$\begin{bmatrix} y^1 \\ \vdots \\ y^P \end{bmatrix} = \begin{bmatrix} A^1 \\ \vdots \\ A^P \end{bmatrix} s_0 + \begin{bmatrix} n^1 \\ \vdots \\ n^P \end{bmatrix} \quad (5)$$

Then, (2) and (3) can be re-written as:

$$x_{t+1} = \eta_t(x_t + \sum_{p=1}^P A^{pT} z_t^p; \tau \sigma_t) \quad (6)$$

$$z_{t+1}^p = y^p - A^p x_{t+1} + \frac{\|x_{t+1}\|_0}{M} z_t^p, \forall p = 1, \dots, P \quad (7)$$

By introducing an intermediate matrix $W_t = [w_t^1, \dots, w_t^P]$ with each column computed by the corresponding sensor as:

$$w_t^p = \begin{cases} x_t + A^{pT} z_t^p, & p = 1 \\ A^{pT} z_t^p, & \text{otherwise} \end{cases} \quad (8)$$

which is similar to that in [4], (6) becomes

$$x_{t+1} = \eta_t(\sum_{p=1}^P w_t^p; \tau \sigma_t) \quad (9)$$

Therefore, DAMP can be divided into two parts: local computation of z_t^p and w_t^p ($p = 1, \dots, P$), and global computation of x_{t+1} and σ_{t+1} , in which transmission of data between sensors is needed. For the latter, a natural approach is to send all the data in w_t^p ($p = 2, \dots, P$) to sensor 1, which induces a high communication cost when N is large. Therefore, how to reduce the communication cost, meanwhile maintaining the same recovery solution as the centralized AMP, is the main focus of this paper.

C. GCAMP Algorithm

Let us denote $v(n)$ as the n -th component of a vector v . According to (9), $x_{t+1}(n) = 0$ if $|\sum_{p=1}^P w_t^p(n)| \leq \beta = \tau \sigma_t$. Therefore, we only need to know all the n s such that $|\sum_{p=1}^P w_t^p(n)| > \beta$ in the global computation. This is similar to Top-K problem in the field of distributed database querying, which is to find the K largest components of $\sum_{p=1}^P w_t^p$. In [9] the three-phase uniform threshold (TPUT) algorithm, an efficient approach to solve the Top-K problem with a known K , is proposed. However, our problem is different from the Top-K problem. First, we do not know how many components of $\sum_{p=1}^P w_t^p$ have magnitude larger than β ; second, TPUT requires $w_t^p(n)$'s to be non-negative, while in our problem, they can be any real numbers. Hence, TPUT cannot be applied in our case. Nevertheless, it does provide some insight on how to design the communication algorithm in distributed systems. Here, we propose the GCAMP algorithm which is shown in Table I.

TABLE I
GCAMP ALGORITHM

Input $w_t^1, \dots, w_t^P, \beta = \tau \sigma_t$;

Step I Set $T = \beta \theta / (P - 1)$, where $\theta \in (0, 1)$ is a tuned parameter;
for sensor $p = 2:P$
 denote $R_p = \{n : |w_t^p(n)| > T\}$;
 send all $(n, w_t^p(n))$ pairs for $n \in R_p$ to sensor 1;
endfor
Step II for sensor 1, define $I_S(x) := 1$ if $x \in S$; 0 o.w.
for $n = 1:N$
 get $S_n := \{p = 2, \dots, P : I_{R_p}(n) = 1\}$ with cardinality m_n ;
 Compute $U(n) = |w_t^1(n) + \sum_{p \in S_n} (w_t^p(n))| + (P - 1 - m_n)T$;
 if $U(n) > \beta$ and $m_n < P - 1$
 broadcast the index n to other sensors;
 endif
endfor
Step III denote $F = \{n : U(n) > \beta, m_n < P - 1\}$;
for sensor $p = 2:P$
 send all $(n, w_t^p(n))$ pairs for $n \in F \setminus R_p$ to sensor 1;
endfor
Step IV for sensor 1, initialize $x_{t+1} = 0$;
for $n \in V := \{n : U(n) > \beta\}$
 Update $x_{t+1}(n) = \eta_t(\sum_{p=1}^P w_t^p(n); \beta)$ by (4);
endfor

Output x_{t+1}

Theorem 1: In each iteration, $U(n)$ is an upper bound of $|\sum_{p=1}^P w_t^p(n)|$ for all n , and the x_{t+1} which GCAMP algorithm obtains (denoted as x_{t+1}^G) is exactly the same as that obtained by the original centralized AMP algorithm (denoted as x_{t+1}^A).

Proof: For any $n = 1, \dots, N$, we have

$$\sum_{p=1}^P w_t^p(n) = w_t^1(n) + \sum_{p \in S_n} w_t^p(n) + \sum_{p \geq 2, p \notin S_n} w_t^p(n) \quad (10)$$

Then, applying the triangle inequality, we have

$$\begin{aligned} |\sum_{p=1}^P w_t^p(n)| &\leq |w_t^1(n) + \sum_{p \in S_n} w_t^p(n)| + |\sum_{p \geq 2, p \notin S_n} w_t^p(n)| \\ &\leq |w_t^1(n) + \sum_{p \in S_n} w_t^p(n)| + (P - 1 - m_n)T = U(n) \end{aligned} \quad (11)$$

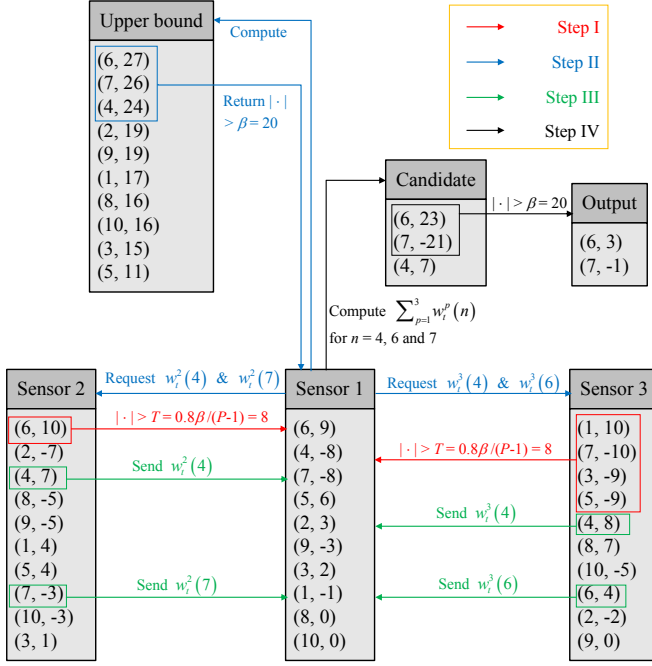


Fig. 1. An example of GCAMP algorithm

$\forall n \notin V$, $x_{t+1}^G(n) = 0$; by (11), $|\sum_{p=1}^P w_t^p(n)| \leq U(n) \leq \beta$, so $x_{t+1}^A(n) = 0$. $\forall n \in V$, $x_{t+1}^G(n) = x_{t+1}^A(n) = \eta_t(\sum_{p=1}^P w_t^p(n); \beta)$. Therefore, $x_{t+1}^G = x_{t+1}^A$.

In Fig. 1, an example is provided to illustrate how GCAMP works, in which each sensor p already sorts $w_t^p(n)$ in descending order of magnitudes, and stores the data in the form of $(n, w_t^p(n))$ pairs ($p = 1, \dots, 3, n = 1, \dots, 10$). Suppose $\beta = 20$ and $\theta = 0.8$, since we have $P = 3$ sensors, we get $T = \beta\theta/(P-1) = 8$. In step I, sensors 2 to P send all $(n, w_t^p(n))$ pairs with $|w_t^p(n)| > T$ to sensor 1. In step II, sensor 1 receives the data, computes upper bounds $U(n)$ for $n = 1, \dots, 10$ and obtains $F = V = \{4, 6, 7\}$. Then sensor 1 broadcasts indices in $n \in F$. In step III, sensor 2 sends $w_t^2(4)$ and $w_t^2(7)$, and sensor 3 sends $w_t^3(4)$ and $w_t^3(6)$ to sensor 1. Finally, in step IV, sensor 1 computes x_{t+1} for $n \in V$ by (8), and outputs the non-zero components of x_{t+1} . Overall, in this example, only 9 data points are sent from other sensors to sensor 1, and the total number of messages is 12 (9 data points plus 3 broadcast requests).

D. Tuning of τ Values

With the GCAMP algorithm, DAMP can be developed. We adopt the tuning framework in [10] to find the optimal value for τ . First, a descending candidate list of candidate values of τ , $\{\tau\}_{l=1}^L := [\tau_{\max}, \dots, \tau_{\max} - (l-1)\Delta\tau, \dots, \tau_{\max} - (L-1)\Delta\tau]$ is generated. Then, for each candidate τ_l , we run iterations in (6) and (7) until x_t and σ_t converge to x_t^* and σ_t^* , and use them as the initial estimates for the iterations using the next candidate τ_{l+1} . We repeat this process until σ_l^* is not decreasing, and get the optimal τ value as well as the final estimate of s_0 . The pseudo code of DAMP algorithm is shown

in Table II.

How to choose the maximum candidate value, i.e., τ_{\max} , is also an interesting problem. In [10], the authors set $\tau_{\max} = \frac{\|A^T y\|_{\infty}}{\sqrt{\|y\|_2^2/M}}$, which $\|\cdot\|_{\infty}$ is the magnitude of the largest-in-magnitude component in a vector. Denote $\tilde{x}_t := x_t + A^T z_t = \sum_{p=1}^P w_t^p$, since at the beginning, $x_0 = 0$ and $z_0 = y$, we have $\tilde{x}_0 = A^T y$ and $\sigma_0 = \sqrt{\frac{\|y\|_2^2}{M}}$. Therefore, $\forall n = 1, \dots, N$, we have $|\tilde{x}_0(n)| \leq \tau_{\max} \sigma_0$. This implies that the optimal value for τ cannot be greater than τ_{\max} . Here, we propose a different approach. According to [11], as $N \rightarrow \infty$, asymptotically each component of $\tilde{x}_t - s_0$ is independent and identically distributed (i.i.d.) random variable, following a $\mathcal{N}(0, \sigma_t^2)$ distribution. Therefore, we can build a $(1 - \alpha)$ confidence interval (CI) $[-z_{\frac{\alpha}{2}} \sigma_t, z_{\frac{\alpha}{2}} \sigma_t]$, where z_{α} is defined such that $\frac{1}{\sqrt{2\pi}} \int_{z_{\alpha}}^{+\infty} \exp(-\frac{t^2}{2}) dt = \alpha$. Hence, $\forall n = 1, \dots, N$, if $s_0(n) = 0$, with probability $1 - \alpha$, $\tilde{x}_t(n)$ will be in the CI; on the other hand, if for some n , $|\tilde{x}_t(n)| > z_{\frac{\alpha}{2}} \sigma_t$, then with probability at least $1 - \alpha$, $s_0(n)$ is a non-zero component. Therefore, we can choose a very small α , and let $\tau_{\max} = z_{\frac{\alpha}{2}}$. For example, we can let $\alpha = 0.0027$ and $\tau_{\max} = z_{\frac{\alpha}{2}} = 3$.

Note that in every iteration involving (6) and (7), after GCAMP returns x_{t+1} , sensor 1 broadcasts non-zero components of x_{t+1} as well as their indices. In DAMP, we tune the optimal τ value in a descending order, which implies a larger threshold $\beta = \tau \sigma_t$ in the beginning. Therefore, different from [3], we have a sparse estimate x_{t+1} even at the first few iterations. Hence, the communication cost for broadcasting x_{t+1} is negligible compared with that of GCAMP. Once knowing x_{t+1} , each local sensor can obtain z_{t+1}^p using (7) and $\sigma_{t+1}^p = \|z_{t+1}^p\|_2$ ($p = 1, \dots, P$). Next, each sensor $p \geq 2$ just sends a scalar σ_{t+1}^p to sensor 1, which needs $P-1$ messages. Then, sensor 1 computes $\sigma_{t+1} = \sqrt{\sum_{p=1}^P (\sigma_{t+1}^p)^2/M}$, updates β and T , and broadcasts the scalar T to other sensors. Overall, GCAMP incurs most of the communication cost in DAMP.

E. Comparison of GCAMP and Modified TA

TA [5] is another popular algorithm solving Top-K problems. Similar to TPUT, TA also requires the knowledge of K and all entries in W_t to be non-negative. Therefore, we propose a modified TA algorithm as in Table III, and let it be a control algorithm for GCAMP.

Theorem 2: In each iteration, Modified TA algorithm also gives exactly the same x_{t+1} as that of original AMP algorithm.

Proof: Modified TA is composed of a series of global summation, where a global summation means computing $|\sum_{p=1}^P w_t^p(n)|$ for some n . N_s is a counter recording the number of global summations. At the very end of one global summation, for each n , either the $(n, w_t^p(n))$ pairs for all p are marked as “sent”; or none of them are marked as “sent”. So we can just say n is marked as “sent” or not. It is easy to show that, $\sum_{p=1}^P |u_p|$ is an upper bound of $|\sum_{p=1}^P w_t^p(n)|$ for all n that have not been marked as “sent”; if $\sum_{p=1}^P |u_p| \leq \beta$, then we have $|\sum_{p=1}^P w_t^p(n)| \leq \beta$ for these n . As the algorithm terminates, we do not lose any non-zero components of x_{t+1} .

TABLE II
DAMP ALGORITHM

Input $\{y\}_{p=1}^P, \{A\}_{p=1}^P, \{\tau\}_{i=1}^L, \text{maxiter}, \epsilon;$

Initialization $x_0 = 0, z_0^p = y^p$ for $p = 1 \cdots P, \sigma_0 = \sqrt{\sum_{p=1}^P \|z_0^p\|_2^2 / M};$
for $i = 1:L$
 for $t = 1:\text{maxiter}$
 for $p = 1:P$
 Compute w_{t-1}^p by (8);
 endfor
 $x_t = \text{GCAMP}(w_{t-1}^1, \dots, w_{t-1}^P, \beta = \tau_i \sigma_{t-1});$
 for $p = 1:P$
 Compute z_t^p by (7);
 endfor
 $\sigma_t = \sqrt{\sum_{p=1}^P \|z_t^p\|_2^2 / M}$
 if $|\sigma_t - \sigma_{t-1}| < \epsilon \sigma_{t-1}$
 $\sigma(\tau_i) = \sigma_t, x(\tau_i) = x_t, z^p(\tau_i) = z_t^p$ for $p = 1 \cdots P;$
 break;
 endif
 endfor
 if $\sigma(\tau_i) > \sigma(\tau_{i-1})$
 $\tau^* = \tau_{i-1}, \sigma^* = \sigma(\tau^*), x^* = x(\tau^*);$
 return;
 else
 $\sigma_0 = \sigma(\tau_i), x_0 = x(\tau_i), z_0^p = z^p(\tau_i)$ for $p = 1 \cdots P;$
 endif
endfor

Output τ^*, σ^*, x^*

Number of Messages: For a set, denote $|\cdot|$ as its cardinality. For GCAMP, the total number of messages is $\sum_{p=1}^P |R_p| + |F| + \sum_{p=1}^P |F \setminus R_p|$; for Modified TA, in each global summation, there are 1 broadcasting message from some sensor to others and $P-1$ incoming messages, so the total number of messages is PN_s . It is easy to check that, for the data set in Figure 1, Modified TA needs $PN_s = 3 \times 9 = 27$ messages, more than twice of that of GCAMP.

III. NUMERICAL RESULTS

A. Performance Measures

Since we have proved that the DAMP algorithm has exactly the same solution as the original AMP, and the recovery accuracy and convergence of AMP has been well studied in literature, it is not necessary to evaluate them again in the paper. Instead, as DAMP is a distributed algorithm, it is important to evaluate the communication cost saved by using GCAMP. So we use the number of messages transmitted as the performance measure, which is widely used in literature [5], [9]. We compare the number of messages used in GCAMP to that in Modified TA. Considering the approach sending all data to sensor 1, which has a total number of messages $N(P-1)$, we define normalized message number (NMN) as

$$\mu_M = \frac{\text{number of messages in computing } x_{t+1}}{N(P-1)} \quad (12)$$

which is $\mu_M = \frac{\sum_{p=1}^P |R_p| + |F| + \sum_{p=1}^P |F \setminus R_p|}{N(P-1)}$ for GCAMP and $\mu_M = \frac{N_s P}{N(P-1)}$ for Modified TA.

TABLE III
MODIFIED TA ALGORITHM

Input $w_t^1, \dots, w_t^P, \beta = \tau \sigma_t;$

Initialization $x_{t+1} = 0, N_s = 0;$
for sensor $p = 1:P$
 sort components of w_t^p in descending order of magnitudes;
 define the sorted vector as s_t^p and $I_t^p(n) := l$ s.t. $w_t^p(l) = s_t^p(n);$
 mark all $(I_t^p(n), s_t^p(n))$ pairs as “unsent”;
endfor
while 1
 for $p = 1:P$, do the following process named global summation
 find the first $(I_t^p(n), s_t^p(n))$ pair marked “unsent” from top;
 set $u_p = s_t^p(n)$, broadcast $(I_t^p(n), u_p)$ to other sensors;
 mark $(I_t^p(n), s_t^p(n))$ as “sent”;
 for sensor $q \neq p$
 store u_p and send $(I_t^p(n), w_t^q(I_t^p(n)))$ to sensor $p;$
 mark $(I_t^p(n), w_t^q(I_t^p(n)))$ as “sent”;
 endfor
 update $x_{t+1}(I_t^p(n)) = \eta_t(\sum_{p=1}^P w_t^p(I_t^p(n)); \beta);$
 number of global summations $N_s = N_s + 1;$
 if $N_s \geq P$ and $\sum_{p=1}^P |u_p| \leq \beta$, or if $N_s \geq N$
 the algorithm terminates;
 endif
endfor
endwhile

Output x_{t+1}

B. Simulation Setup

Our focus is not to investigate large-scale problems, but to develop distributed algorithms and evaluate their efficiency in reducing communication costs. Nevertheless, we still use a considerably large $N = 5000$, and choose κ from $[0.1, 0.5]$, ρ from $[0.1, 0.3]$, which leads to $M = N\kappa$ in $[500, 2500]$ and $K = M\rho$ in $[50, 750]$. The problem scales used in our paper is larger than those used in other DCS publications [4]. The number of sensors P is within $[5, 50]$. The sensing matrix A with i.i.d. entries $\sim \mathcal{N}(0, \frac{1}{M})$ is partitioned into P parts with each sensor having a $(M/P) \times N$ submatrix. Each component of s_0 is i.i.d. drawn from

$$f_X(x) = \kappa \rho G(x) + (1 - \kappa \rho) \delta(x) \quad (13)$$

where $G(x)$ is the probability density function (pdf) of the standard Gaussian distribution and $\delta(x)$ is the Dirac Delta function. The measurements of s_0 are corrupted by an additive noise $n \sim \mathcal{N}(0, \sigma^2 I_M)$ and σ is the standard deviation with a value in $[0.01, 0.1]$. The parameter θ in GCAMP is set to 0.8. Regarding the tuning procedure for optimal τ values, we make a candidate list for τ of length 11, starting from 3 with a step -0.2; for each candidate, the convergence criteria is $|\sigma_t - \sigma_{t-1}| < 0.01 \sigma_{t-1}$. We compare $\bar{\mu}_M$ defined as μ_M averaged over iterations based on 100 Monte-Carlo runs.

C. Performance Evaluation

We evaluate $\bar{\mu}_M$ in three settings: I) fix $\sigma = 0.02$ and $P = 10$, and change the values of κ and ρ ; II) fix $\kappa = 0.2$, $\rho = 0.1$ and $P = 10$, and change the values of σ ; III) fix $\kappa = 0.2$,

$\rho = 0.1$ and $\sigma = 0.02$, and change the values of P . Tables IV, V and VI show the corresponding numerical results for I), II) and III) respectively. In the tables, the former entry in each pair inside the parentheses denotes $\bar{\mu}_M$ for GCAMP, and the latter denotes that for Modified TA. It is clear that in each case, GCAMP outperforms Modified TA significantly. Modified TA always uses more messages than $N(P-1)$ except for the case $P = 5$, while GCAMP can save the number of messages from 22.7% to 48.2%. Fig. 2 gives the cumulative distributions of μ_M in each iteration for GCAMP and Modified TA under 4 different scenarios: 1) $\kappa = 0.2, \rho = 0.1, \sigma = 0.02, P = 5$; 2) $\kappa = 0.2, \rho = 0.1, \sigma = 0.02, P = 10$; 3) $\kappa = 0.2, \rho = 0.1, \sigma = 0.01, P = 10$; 4) $\kappa = 0.3, \rho = 0.1, \sigma = 0.02, P = 10$. It provides us much more detailed information on the distribution of μ_M for each algorithm. It is clear that under each scenario, Modified TA uses more than $N(P-1)$ messages in at least 33.4 % of the total iterations; while GCAMP never uses more than $0.91N(P-1)$ messages in any iteration, and among more than 95% of the total iterations, it just uses $[40\%, 80\%] \times N(P-1)$ messages, that is, it can save 20% ~ 60% of the messages with probability at least 95%.

TABLE IV
 $\bar{\mu}_M$ FOR GCAMP AND MODIFIED TA WITH DIFFERENT κ AND ρ

	$\kappa = 0.1$	0.2	0.3	0.4	0.5
$\rho=0.10$	(0.547, 1.101)	(0.567, 1.103)	(0.573, 1.103)	(0.587, 1.103)	(0.589, 1.103)
0.15	(0.621, 1.108)	(0.616, 1.106)	(0.632, 1.107)	(0.635, 1.107)	(0.639, 1.106)
0.20	(0.659, 1.108)	(0.667, 1.108)	(0.672, 1.108)	(0.691, 1.109)	(0.684, 1.108)
0.25	(0.651, 1.107)	(0.689, 1.109)	(0.707, 1.109)	(0.725, 1.109)	(0.731, 1.109)
0.30	(0.632, 1.108)	(0.690, 1.109)	(0.737, 1.109)	(0.751, 1.110)	(0.755, 1.110)

TABLE V
 $\bar{\mu}_M$ FOR GCAMP AND MODIFIED TA WITH DIFFERENT σ

$\sigma = 0.01$	0.02	0.03	0.04	0.05
(0.564, 1.103)	(0.567, 1.103)	(0.574, 1.104)	(0.576, 1.104)	(0.582, 1.104)
$\sigma = 0.06$	0.07	0.08	0.09	0.1
(0.583, 1.104)	(0.589, 1.104)	(0.590, 1.104)	(0.592, 1.105)	(0.590, 1.105)

TABLE VI
 $\bar{\mu}_M$ FOR GCAMP AND MODIFIED TA WITH DIFFERENT P

$P = 5$	10	15	20	25
(0.518, 0.941)	(0.567, 1.103)	(0.623, 1.071)	(0.664, 1.053)	(0.694, 1.042)
$P = 30$	35	40	45	50
(0.717, 1.034)	(0.735, 1.029)	(0.751, 1.026)	(0.763, 1.023)	(0.773, 1.020)

IV. CONCLUSION

Assuming the sparsity of the original signal to be unknown, the DAMP approach has been developed for performing compressed sensing in distributed sensor networks, consisting a series of local and global computations. We proposed the

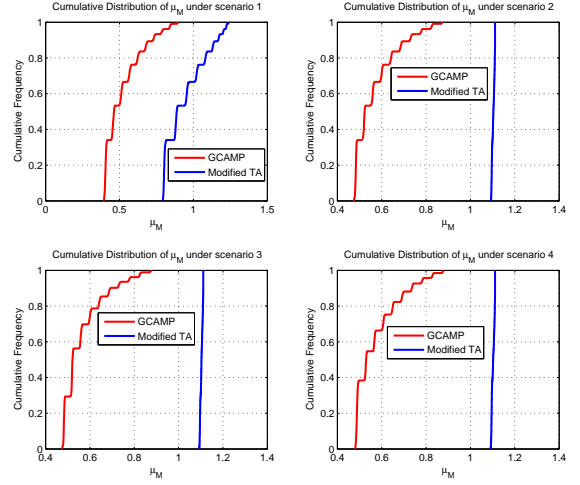


Fig. 2. Cumulative distributions of μ_M for GCAMP and Modified TA
GCAMP in the stage of global computation to reduce the number of messages per iteration, and proved theoretically that DAMP based on GCAMP has exactly the same solution as the original AMP. Meanwhile, we modified TA algorithm so that it can be used in DAMP, which also has exactly the same solution as the original AMP, and used it as the control algorithm for GCAMP in evaluating the communication cost savings. Numerical results demonstrated that GCAMP based DAMP outperforms Modified TA based DAMP significantly, and is very efficient in reducing communication costs.

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